LITERATURE CITED

1. O. A. Gerashchenko, Principles of Heat Measurement [in Russian], Naukova Dumka (1971).
2. B. Douglas, Instrum. Contr. Syst., 45, No. 5 (1975).
3. G. N. Dul'nev and N. V. Pilipenko, Inzh.-Fiz. Zh., 29, No. 5 (1975).
4. G. Doetsch, Guide to the Application of Laplace and Z Forms, 2nd ed., Van Nostrand Reinhold.
5. M. L. Krasnov, Integral Equations [in Russian], Nauka, Moscow (1975).
6. G. A. Korn and T. M. Korn, Manual of Mathematics, McGraw-Hill (1967).
7. A. V. Lykov, Theory of Heat Conduction [in Russian], Vysshaya Shkola, Moscow (1967).

DETERMINING THERMAL RESISTANCE OF CONTACT BETWEEN FINISHED WAVY METALLIC SURFACES

V. M. Popov

UDC 536.21

A design formula for determining the thermal resistance of a contact is obtained using functions describing the relief of real wavy surfaces.

Previously completed comparative experimental investigations into the thermal contact resistance (TCR) of metallic contacts between flat-rough and wavy surfaces [1, 2] have established a significant increase in the TCR for the latter for virtually identical grades of surface finish, while an increase in the height of the waves in the surface causes a marked increase in the TCR. At the same time, the theoretical model given in [2, 3] for determining the thermal contact resistance of wavy surfaces is to a certain extent idealized, inasmuch as a homogeneous distribution of the waves by height relative to a standard plane is taken as one of the basic premises. An analysis of profilograph traces from finished metallic surfaces shows that real surfaces represent in most cases a set of waves of a usually spherical or ellipsoidal form with a constant radius subject to a normal (Gaussian) law of distribution by height [4].

Let us examine a contact couple with a wavy surface [2]. In general, the thermal contact conditions presuppose a temperature drop common to all macrocontacts:

$$\Delta T_C = \frac{Q}{2\bar{\lambda}_M a}\varphi.$$

Hence for all macrocontacts the following equality obtains:

$$2\bar{\lambda}_M \Delta T_C = \frac{Q}{a/\varphi}.$$

According to the last equality, the total heat flow is divided as it passes through the individual macrocontacts, i.e., we have the following relation:

$$\frac{Q_1}{a_1/\varphi_1} = \frac{Q_2}{a_2/\varphi_2} = \ldots = \frac{Q_m}{a_m/\varphi_m};$$

hence with an unvariable φ

Voronezh Wood Technology Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 32, No. 5, pp. 779-785, May, 1977. Original article submitted February 24, 1976.

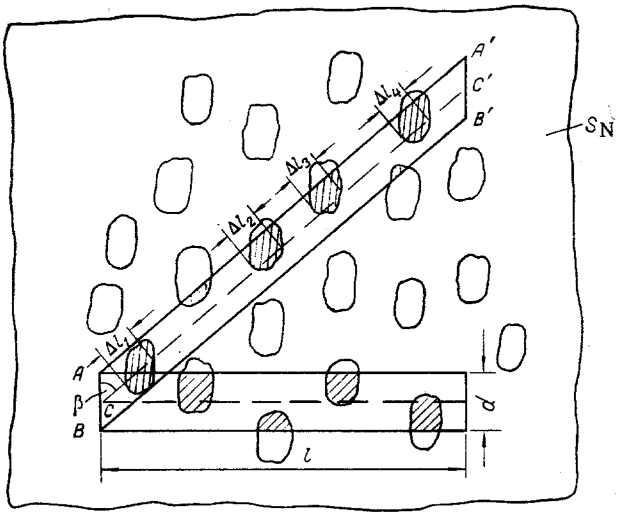

Fig. 1. Diagram of cross section of wavy surface through a plane parallel to the geometrical plane.

$$Q_i = Q\frac{a_i'\varphi_i}{\sum\limits_{k=1}^{m} a_C'\varphi_C} = Q\frac{a_i}{\sum\limits_{k=1}^{m} a_C}.$$

The model examined for a thermal contact between two solids presupposes the existence (at a distance from the interface) of thermal channels with cross sections S_{N_1}, S_{N_2}, etc., through which the heat flux is fed to the appropriate macroscopic regions of the contact, i.e.,

$$S_{N_i} = S_N\frac{Q_i}{Q} = S_N\frac{a_i'\varphi_i}{\sum\limits_{k=1}^{m} a_C'\varphi_C} = S_N\frac{a_i}{\sum\limits_{k=1}^{m} a_C}.$$

It is possible, using the relations given, to express the thermal resistance for one or several macrocontacts in the form

$$R = \frac{\Delta T_C}{Q/S_N} = S_N\sum_{i=1}^{n}\frac{\Delta T_C}{Q_i} = \frac{S_N}{2\bar{\lambda}_M}\sum_{i=1}^{n}\frac{\varphi_i}{a_i} = \frac{\varphi}{2\bar{\lambda}_M\sum\limits_{i=1}^{n} a_i}S_N = \frac{\pi}{2}\frac{\left|\frac{\bar{L}}{2}\right|\varphi}{2\bar{\lambda}_M\eta_2^{1/2}}. \tag{1}$$

Dependence (1) can be realized for a contact between real wavy surfaces given information on the magnitude of $\Sigma a_i/S_N$ or η_2 taking into account the probability of projections of the waves coming into contact with the plane.

Let us examine a profilograph trace on a line of length l (Fig. 1) plotted from a wavy surface of area S_N with a stationary profile at an angle of β to the direction of finishing. For an arbitrarily selected level of the profilograph trace the values for the length of the segments of the secant along a given direction are equal to, respectively, Δl_1, Δl_2, ..., Δl_n. If the AA' and BB' lines are continued parallel to CC' for a distance of $d/2$, significantly less compared with the dimensions of S_N and S_C, then by virtue of the stationary nature of the profile the relative contour area of the wave cross sections can be represented as

$$\eta_2 = \frac{S_C}{S_N} = \sum_{i=1}^{n}\frac{\Delta S_{Ci}}{\Delta S_N} = d\sin\beta\sum_{i=1}^{n}\frac{\Delta l_i}{ld\sin\beta} = \frac{\sum\limits_{i=1}^{n}\Delta l_i}{l}. \tag{2}$$

Here ΔS_N and ΔS_{Ci} are, respectively, the areas of the AA' and BB' zone and of the wave cross sections in the zone.

If the contour areas S_{C_i} are represented by circles with a radius a_i and if we assume that $d \ll a_i$, then the number of lines intersecting the area S_{C_i} in the first approximation is equal to $2(a_i/d)$. Hence the total number of intersections of all the S_{C_i} areas by lines parallel to AA' is $2(a_1 + a_2 + \ldots + a_n)/d$. For all the lines the derived number of intersections N per unit length of the line l lying within the limits of the nominal contact surface under examination is described by the relation

$$\frac{N}{l} = \lim_{d \to 0} \left[\frac{2(a_1 + a_2 + \ldots a_n)/d}{S_N/d} \right] = \frac{2\sum\limits_{i=1}^{n} a_i}{S_N} = \frac{2\eta_2}{\pi/|\overline{L/2}|}. \tag{3}$$

The mutual transformation of relations (1)-(3) enables us to establish the relationship between the thermal resistance, the number of intersections, and the length of the tracks at the contact sites:

$$\frac{\varphi}{R\lambda_M} = \frac{N}{l} = \frac{2}{\pi} \frac{\sqrt{\dfrac{\sum\limits_{i=1}^{n} \Delta l_i}{l}}}{|\overline{L/2}|}. \tag{4}$$

It follows from (4) that in order to determine the TCR for systems with wavy surfaces it is essential to establish the relationship between N/l and $\Sigma\Delta l_i/l$ in each individual case of contact. The most reliable information on the relationship between N/l and $\Sigma\Delta l_i/l$ is provided by an analysis of mutually superimposed profiles. Taking into account the difficulties related to the operation of profile selection, however, the establishment of the relationship between N/l and $\Sigma\Delta l_i/l$ using functions characterizing the relief of real surfaces is of practical interest.

We introduce the assumption that every wavy surface coming into contact with another has certain probability criteria describing its profile. This assumption presupposes the existence of functions characterizing the profile of the surfaces $H(l)$ and $f(l)$, the distribution of the crests of the waves on the $F(H)$ surface, and the probability functions for the slope of the $F'(H')$ profile. Moreover, we assume that the random process $H(l)$ is stationary and the wave crest distribution is normal, i.e.,

$$F(H) = \frac{1}{H_{rms}\sqrt{2\pi}} \cdot l^{-\frac{H^2}{2H_{rms}^2}}. \tag{5}$$

Here H is measured from the center line, and the root-mean-square deviation of the wave crests is

$$H_{rms} = \int_{-\infty}^{+\infty} H^2 F(H)\, dH.$$

By analyzing the profile of the surface in Fig. 2, the expected number of waves per unit length of the profile above a certain level H_0 can be sought. For this we introduce a step function $\Phi(i) = 1$ for $i > 0$, and $\Phi(i) = 0$ for $i < 0$. By differentiating $\Phi(H - H_0)$, we obtain

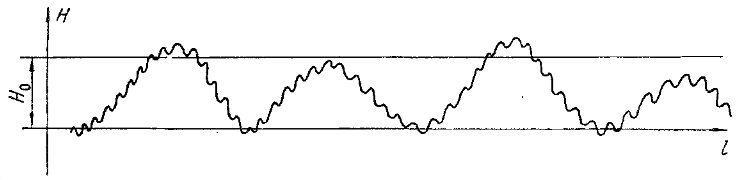

Fig. 2. Profile of wavy surface.

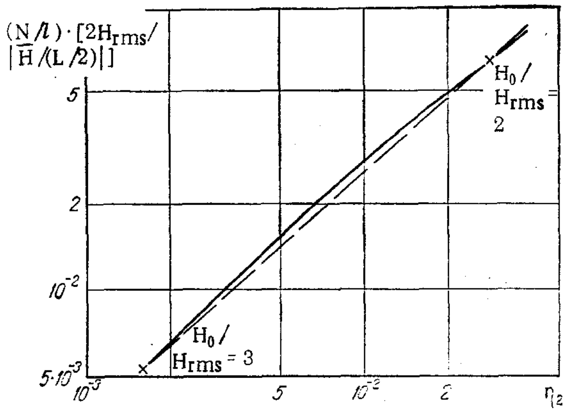

Fig. 3. Relationship between number of contacts of profile of wavy surface and relative contour area of contact.

$$\frac{d}{dl}[\Phi(H-H_0)] = H'\Delta(H-H_0),$$

where $\Delta(H - H_0)$ is a single pulse satisfying the relationship $\int_{-\infty}^{+\infty}\Delta(H-H_0)\,dH = 1$ and tending toward zero, with the exception of $H = H_0$.

If for $l_1 < l < l_2$ the $H(l)$ function has only a value H_0, then $\int_{l_1}^{l_2}|H'|\Delta(H-H_0)\,dl = 1.$

Hence the number of wave intersections per unit length of the profile is expressed as

$$N/l = \lim_{l\to\infty}\frac{1}{2l}\int_0^l |H'|\Delta(H-H_0)\,dl$$

or, taking into account the wave distribution of the profile from the distribution of the slopes of the profile,

$$N/l = \frac{1}{2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}|H'|\Delta(H-H_0)F(H)F'(H')\,dH\,dH' = \frac{F(H)}{2}\int_{-\infty}^{+\infty}|H'|F'(H')\,dH'. \quad (6)$$

The mutual transformation of the relationships (5) and (6) gives

$$\frac{2H_{rms}\frac{N}{l}}{|\overline{H/L/2}|} \cdot \frac{e^{-\frac{H_0^2}{2H_{rms}^2}}}{\sqrt{2\pi}}, \quad (7)$$

where $|\overline{H/L/2}| = \int_{-\infty}^{+\infty}|H'|F'(H')\,dH$ is the derived value for the absolute slope of the profile of the wavy surface.

According to the determination of F(H),

$$\frac{\sum_{i=1}^{n}\Delta l_i}{l} = \int_{H_0}^{\infty}F(H)\,dH. \quad (8)$$

Eliminating H_0 from (7) and (8), we obtain a relationship relating the number of intersections to the segment in the contact or to the relative contour area of the contact:

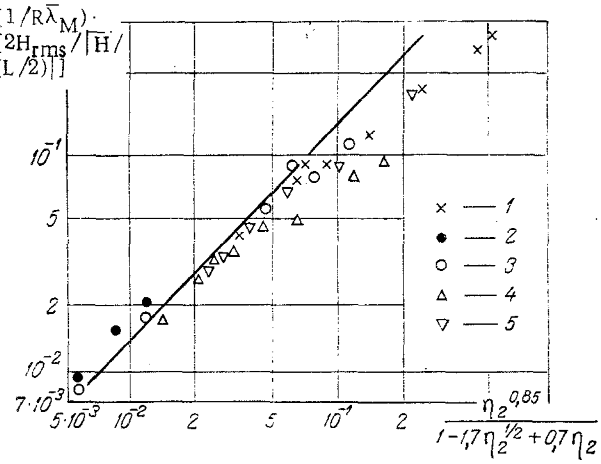

Fig. 4. Dependence of dimensionless conductivity on relative area of actual contact of wavy surfaces (in vacuum): 1) D16T—D16T, H_{rms} = 5.1 μ, $|\bar{H}/(L/2)|$ = 0.0095; 2) same for repeated loadings; 3) 2Kh13—2Kh13, H_{rms} = 13.8 μ, $|\bar{H}/(L/2)|$ = 0.0162; 4) steel 45—steel 45, H_{rms} = 4.5 μ, $|H/(L/2)|$ = 0.0111; 5) steel 45—steel 45, H_{rms} = 4 μ, $|\bar{H}/(L/2)|$ = 0.0088. Curve plotted from formula (11).

$$\frac{N}{l}\;\frac{2H_{rms}}{\left|H\Big/\frac{L}{2}\right|}=K\left(\frac{\sum\limits_{i=1}^{n}\Delta l_i}{l}\right)=K\eta_2. \tag{9}$$

The relationship obtained is interpreted graphically in logarithmic coordinates in Fig. 3. Here the approximation is shown in the form of a straight (dashed) line for the region of H_0/H_{rms} values of practical importance which is described by the relation

$$\frac{N}{l}\;\frac{2H_{rms}}{\left|\frac{\bar{H}}{L/2}\right|}=1.3\eta_2^{0.85}. \tag{10}$$

Using the relation (4) and the value of the coefficient of constriction φ [2], the latter expression is transformed into

$$\frac{1}{R\bar{\lambda}_{M}}\cdot\frac{2H_{rms}}{\left|\frac{\bar{H}}{L/2}\right|}=1.1\left(\frac{\eta_2^{0.85}}{1-1.7\eta_2^{1/2}+0.7\eta_2}\right). \tag{11}$$

The results of a calculation by formula (11) are compared in Fig. 4 with experimental data for wavy surfaces from which preloading profilograph traces are plotted and the relationships between H_{rms} and $|\bar{H}/(L/2)|$ are found. An analysis of the data in Fig. 4 shows that the best agreement between theory and experiment is observed for low initial load values, when the contact conditions correspond to the conditions in the problem posed. This fact indicates that the exchange of heat through the zone of wavy surface contact is dependent to a significant extent on the presence of maximal wave projections.

The quantities H_{rms} and $|\bar{H}/(L/2)|$ involved in (11) are found from geometrical measurements. The relative area of contact of the wavy surfaces η_2 is determined experimentally by superimposing profiles or using dependences (5), (8), and (9) for a known distribution of wave crests on the F(H) surface.

The calculation dependence (11) obtained can be used to determine the TCR of systems with two wavy surfaces if it is assumed that $H(l) = H_1(l) + H_2(l)$ and H_0 is taken as the distance between the mean planes of the surfaces. The root-mean-square deviation of the wave heights and the value derived for the slope of the wave profile is expressed in this case as follows:

$$H_{rms} = \sqrt{H^2_{rms_1} + H^2_{rms_2}};$$

$$\left|\frac{\bar{H}}{L/2}\right| = \sqrt{\left|\frac{\bar{H}_1}{L/2}\right|^2 + \left|\frac{\bar{H}_2}{L/2}\right|^2}.$$

NOTATION

R, thermal resistance; Q, heat flux; $\lambda_M = 2\lambda_{M_1}\lambda_{M_2}/(\lambda_{M_1} + \lambda_{M_2})$, derived thermal conductivity of materials of two contacting bodies; a, mean radius of macrocontact area; S_N, S_C, nominal and contour areas of contact, respectively; L, pitch of wave; H, height of wave; H_{rms}, root-mean-square height of wave; $\eta_2 = S_C/S_N$, relative area of contact; φ, coefficient of constriction. Indices: 1 and 2, materials of contacting bodies.

LITERATURE CITED

1. V. M. Popov, Heat Exchange in Contact Zone of Detachable and Nondetachable Connections [in Russian], Énergiya, Moscow (1971).
2. V. M. Popov, Inzh.-Fiz. Zh., 27, No. 5 (1974).
3. M. Yovanovich, Intern. J. Heat Mass Transfer, 12, No. 11 (1969).
4. I. V. Kragel'skii, Friction and Wear [in Russian], Mashinostroenie, Moscow (1968).